Linear Algebra and Geometry 1

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In this short recap of the class on linear algebra and geometry I try to mention the important key concepts and their possible applications. For each chapter of the class a few key words and competences are pointed out.

It is you job to assure that you are familiar with the definitions, operations, calulations and applications.

Numbers and Proofs: Key Topics

- types of numbers: natural, integer, rational, real, complex
- notation: summation, products
- proof: direct, by contradiction, by recurrence

Numbers and Proofs: Competences

With this chapter you should

- be able to distinguish different types of numbers
- know the relation between rational numbers and their decimal representation.
- known why $\sqrt{13}$ is not a rational number.
- be able to work with the summation symbol ($\sum$) and the product symbol ($\prod$).
- be able to compute values of arithmetic and geometric sums.
- know the structure of a proof.

Complex Numbers: Key Topics

- arithmetic operations with complex numbers and their geometric interpretation.
- Eulers formula

$$e^{i\alpha} = \cos\alpha + i\,\sin\alpha$$

and the polar representation

$$z = a + i\,b = |z|\; e^{i\phi}$$

- roots of complex numbers
- complex impedance of a system of resistors, capacitors and inductances.

$$R \longrightarrow R \quad , \quad C \longrightarrow \frac{1}{i\,\omega\,C} \quad , \quad L \longrightarrow i\,\omega\,L$$

Complex Numbers: Competences

With this chapter you should

- master real and complex numbers.
- be able to perform arithmetic operations with complex numbers, algebraically and geometrically.
- know about the complex impedance of resistors, capacitors and inductances.
- be able to compute the complex impedance of small system or resistors, capacitors and inductances.

Vectors and Matrices: Key Topics

- algebraic definition of vectors and their arithmetic operations
- algebraic definition of matrices and their arithmetic operations
- multiplication of matrices, inverse matrix, systems of linear equations
- linear regression
- geometric optics

Vectors and Matrices: Competences

With this chapter you should

- master algebraic definition of vectors and their arithmetic operations
- master algebraic definition of matrices and their arithmetic operations
- be able to use multiplication of matrices, inverse matrices and systems of linear equations.
- be able to set up and solve linear regression problems.
- be able to use matrices to represent geometric optics questions.

Systems of Linear Equations: Key Topics

- what is a system of linear equations
- exactly one, infinitely many or no solutions
- matrix representation and the algorithm of Gauss
- row operations, row reduction
- solutions of systems in reduced form
- homogeneous and inhomogeneous systems

Systems of Linear Equations: Competences

With this chapter you should

- be able to recognize, set up and solve systems of linear equations.
- be able to determine and describe all solutions of linear systems of equations, homogeneous and inhomogeneous.
- be able to use the matrix representation of a linear system and then apply the algorithm of Gauss to determine all solutions.
- master row operations and reduction to row echelon form.
- be able to read of solutions of linear system from the reduced row echelon form.

Matrices and LU-Factorization: Key Topics

- elementary transformations and row operations
- the algorithm of Gauss and LU-factorization
- solving linear systems with help of the LU-factorization
- operation count and memory demand to solve linear systems

Matrices and LU-Factorization: Competences

With this chapter you should

- be able to apply matrix operations by hand and with your calculator.
- recognize elementary matrices and their action on matrices by multiplication.
- know the tight connections between elementary matrices, LU–factorization and the algorithm of Gauss.
- be able to determine the inverse matrice of a matrix of size 3×3 or 4×4 by hand.
- apply all of the above operations fast and reliably with you pocket calculator.
- be able to solve special systems of linear equations with you calculator, i.e. no solution, one solution or infinitely many solutions.
- be able to estimate the operation count, computation time and memory requirement for solving linear systems.

Vectors: Key Topics

geometry $\longleftrightarrow$ algebra

- geometric definition of vectors and their arithmetic operations: addition, subtraction, multiplication by scalar
- cartesian coordinates
- vectors in a plane
  - scalar product of two vectors
  - vectors in the plane, equations of straight lines
  - equations of circles
- vectors in space
  - scalar product and vector product
  - triple product and volumes
  - (equations of planes)
  - (equations of spheres)

Vectors: Competences

With this chapter you should

- be able to visualize the basic vector operations in the plane and in space.
- reliable and fast when calculating with vectors in plane and space.
- be able to work with equations of straight lines in different forms: standard form, parameter form, Hesse form.
- be able to work with circles.
- be able to compute vectors products and use the triple product to determine volumes.
- (be able to work with equations of planes in space.)
- (be able to work with equations of spheres.)